# DY 8: Focus Session: Nonlinear Dynamics and Stochastic Processes – Advances in Theory and Applications II

Deterministic chaos and stochastic processes are often seen as opposites. However, not only do they share the aspect of being mechanisms for irregular temporal fluctuations, but more importantly, the theory of stochastic processes has helped to understand and quantify many aspects of chaos. Deterministic diffusion, intermittency, long-range temporal correlations can be generated by simple deterministic systems, but are conveniently characterized by concepts of stochastic processes. In both classes of systems, the inclusion of time delays in feedbacks through memory kernels has introduced additional phenomena, and more recently non-normalizable distributions have been found as causes of ageing. While a unified approach to chaos and stochastics is fascinating and satisfying from a theoretical point of view, it also has surprisingly strong application relevance. Examples include the study of turbulence, the nonlinear dynamics of wind turbines, industrial processes for metal milling and turning, laser dynamics, cardiac dynamics, and neuronal systems.

Organized by Robert Magerle (Chemnitz) and Holger Kantz (Dresden)

Time: Monday 15:00–18:30 Location: H43

**Invited Talk** DY 8.1 Mon 15:00 H43

**Spatio-temporal pattern formation in time-delayed optical systems** — •Svetlana Gurevich — Institute for Theoretical Physics, University of Münster, Germany

Control and engineering of complex spatio-temporal patterns in high-dimensional non-equilibrium systems has evolved as one of the central issues in applied nonlinear science. However, real-world complex systems can be strongly influenced by time delays due to unavoidable finite signal propagation speeds and time-delayed dynamical systems have proven to be a fertile framework for the modeling of nonlinear phenomena. Nonlinear laser dynamics is one of the fields where time-delayed system are frequently employed to model the arising complex dynamics. In this talk, we will explore time-delayed models to describe the behavior of ultrashort pulses in optical micro-cavities, highlighting their potential for the formation of spatio-temporal structures.

DY 8.2 Mon 15:30 H43

**Thermo-optical excitations and mixed-mode oscillations in an injected Kerr microcavity** — •Elias Koch[1], Julien Javaloyes[2], and Svetlana Gurevich[1,3] — [1]Institute for Theoretical Physics, University of Münster, Wilhelm-Klemm-Str. 9, 48149 Münster, Germany — [2]Departament de Fisica, Universitat de les Illes Balears & IAC-3, Cra. de Valldemossa, km 7.5, E-07122 Palma de Mallorca, Spain — [3]Center for Nonlinear Science (CeNoS), University of Münster, Corrensstraße 2, 48149 Münster, Germany

We study the dynamics of a vertically emitting micro-cavity containing a Kerr nonlinearity that is subjected to detuned optical injection. To this end, we present an extended model that allows investigation of the influence of cavity heating, which shifts the microcavity resonance and thus the detuning on a slow time scale. As a consequence of this scale separation, we uncover a canard scenario featuring dark and bright excitations, as well as mixed-mode oscillations that can be manipulated by tuning the injection amplitude and frequency. When the microcavity is coupled to a long external feedback loop, subjecting it to strong time-delayed optical feedback, we can examine the additional influence of the time delay on excitability dynamics, as well as the impact of thermal effects on preliminary studies.

DY 8.3 Mon 15:45 H43

**Back to the future: Fermi–Pasta–Ulam–Tsingou recurrence in a time-delayed system** — •Jonas Mayer Martins[1], Elias Koch[1], Julien Javaloyes[2], and Svetlana V. Gurevich[1] — [1]Institute for Theoretical Physics, University of Münster, Wilhelm-Klemm-Str. 9 and Center for Nonlinear Science (CeNoS), University of Münster, Corrensstrasse 2, 48149 Münster, Germany — [2]Departament de Física and IAC-3, Universitat de les Illes Balears, C/ Valldemossa km 7.5, 07122 Palma de Mallorca, Spain

We demonstrate Fermi–Pasta–Ulam–Tsingou (FPUT) recurrence, a surprising quasi-periodicity of certain spatially extended systems, in a time-delayed system. Although the bi-Riccati system that we study is not integrable, we find in the long-delay limit that its normal form is a partial differential equation approximating the integrable Korteweg–de Vries (KdV) equation, prominently known to exhibit FPUT recurrence. Our results underscore the analogy between spatially extended and time-delayed systems.

DY 8.4 Mon 16:00 H43

**Momentum space induced complex billiard dynamics** — •Lukas Seemann, Jana Lukin, and Martina Hentschel — Institut für Physik, TU Chemnitz, Deutschland

While billiard models have always been a paradigm to study nonlinear dynamics, their class has been enriched by realistic models such as optical cavities, ballistic quantum dots, or graphene systems over the past decades. The originally hard billiard walls are replaced by confinement through total internal reflection of light or potential wells trapping electrons. They are well-known model systems in the field of mesoscopic physics, quantum chaos, and wave-ray correspondence exhibiting a broad range of dynamical behavior, ranging from regular and mixed to purely chaotic dynamics depending on their geometric shape. However, employing their material-specific properties allows one to influence and even control their complex dynamics in more ways. Using an anisotropy (as for electrons in bilayer graphene systems), one can induce chaotic motion even in a circularly shaped cavity [1]. We develop a ray tracing algorithm for anisotropic media and illustrate how anisotropy affects the billiards dynamics in real and phase space. In particular we show how deformation away from the circular shape in real and momentum space, changes the phase space structure and can be optimized to the formation of large stable island [2] that we quantify using Lyapunov exponents.

[1] L. Seemann, A. Knothe, M. Hentschel, Phys. Rev. B 107, 205404 (2023)

[2] L. Seemann, A. Knothe, M. Hentschel, NJP 26, 10 (2024)

DY 8.5 Mon 16:15 H43

**Stochastic Properties of Musical Time Series: Measuring Musical Variability** — Corentin Nelias and •Theo Geisel — Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany

Music philosophers and psychologists have argued that emotions and meaning in music depend on an interplay of expectation and surprise. We aimed to quantify the variability of musical pieces empirically by considering them as correlated dynamical processes. Using a multi-taper method we determined power spectral density (PSD) estimates for more than 550 classical compositions and jazz improvisations down to the smallest possible frequencies [1]. The PSDs typically follow inverse power laws ($1/f^{\beta}$-noise) with exponents near $\beta$=1 for classical compositions, yet only down to a cutoff frequency, where they end in a plateau. Correspondingly the pitch autocorrelation function exhibits slow power law decays only up to a cutoff time, beyond which the correlations vanish abruptly. We determined cutoff times between 4 and 100 quarter note units serving as a measure for the degree of persistence and predictability in music. They tend to be larger in Mozart's compositions than in Bach's, which implies that the anticipation and expectation of the musical progression tends to last longer in Mozart's than in Bach's compositions

[1] C. Nelias, T. Geisel, Nature Comm. 15, 9280 (2024)

**15 min. break**

**Invited Talk** DY 8.6 Mon 16:45 H43

**Nonlinear dynamics and time delays in metal cutting** —

•Andreas Otto — Fraunhofer Institute for Machine Tools and Forming Technology IWU, Reichenhainer Str. 88, 09126 Chemnitz, Germany

Since products and consequently production processes in manufacturing industry becomes more and more individual, an agile and robust process design is important for minimizing costs while guaranteeing high product quality. A fundamental understanding of the underlying physics of the manufacturing processes is essential for finding sweet spots of high productivity with optimal quality of the produced part.

In this talk, we present some examples from metal cutting, where especially the complex behavior of mechanical vibrations also known as chatter lead to undesired noise, tool wear and scraped parts. We show that mechanical vibrations at machine tools can be described by nonlinear delay differential equations and in many situations the time delay is, in addition, distributed, time-varying or state-dependent. Some recent developments for the prediction of the stability of metal cutting processes with respect to chatter vibrations and applications for online chatter detection are presented.

DY 8.7 Mon 17:15 H43

**Stability of power grids concerning strong perturbations - tropical cyclones and increasing resilience** — •Jürgen Kurths — Potsdam Institute for Climate Impact Research, Telegraphenberg, 14473 Potsdam

The infrastructure of our modern society is efficient but also sensitive concerning strong perturbations, as terrorist attacks on the cybersystem or extreme climate events. An important part of modern infrastructure are power grids, which are characterized by multistability. For them, the strongly ongoing transition to distributed renewable energy sources leads to a proliferation of dynamical actors. The desynchronization of a few or even one of those would likely result in a substantial blackout. We discuss the concept of basin stability covering strong perturbations and identify most vulnerable motifs in power grids. To consider the vulnerability of power grids against extreme wind loads, we combine a detailed model of of extreme events, and a cascadable model of the transmission network to provide a holistic co-evolution model to consider wind-induced failures of transmission lines in the Texan electrical network.

DY 8.8 Mon 17:30 H43

**Critical demand in a stochastic model of flows in supply networks** — •Yannick Feld[1] and Marc Barthelemy[1,2] — [1]Université Paris-Saclay, CNRS, CEA, Institut de Physique Théorique, 91191 Gif-sur-Yvette, France — [2]Centre d'Analyse et de Mathématique Sociales (CNRS/EHESS) 54 Avenue de Raspail, 75006 Paris, France

Supply networks are essential to modern production, yet their critical properties remain understudied. This talk presents a stochastic model with random production capacities that allows us to investigate material flow to a root node, focusing on the network topology and buffer stocks. We examine the critical demand at the root, where unsatisfied demand diverges. Without stocks, minimal production governs behavior, and topology is mostly irrelevant. Buffer stocks introduce memory, significantly altering the dynamics. Now the topology is crucial, with local connectivity proving beneficial.

DY 8.9 Mon 17:45 H43

**Generalizations of laminar chaos** — •David Müller-Bender — Institute of Physics, Chemnitz University of Technology, 09107 Chemnitz, Germany

Laminar chaos was originally discovered in scalar dynamical systems with a large periodically time-varying delay [Phys. Rev. Lett. 120, 084102 (2018)]. This demonstrated how drastically a temporal modulation of the delay can change the dynamics of a system as laminar chaos is an extremely low-dimensional dynamics compared to turbulent chaos, which is observed in such systems with constant delay. In this talk, I give an overview of recent generalizations of laminar chaos to systems with quasiperiodic [Phys. Rev. E 107, 014205 (2023)], random and chaotic delay time modulation. Using a connection to spatially disordered circle maps [Phys. Rev. E 106, L012202 (2022)], it is found that short-time correlated random and chaotic delays lead to low-dimensional generalized laminar chaos in almost the whole delay parameter space spanned by the mean delay and the delay amplitude. This is in stark contrast to the case of a constant delay, where only high-dimensional turbulent chaos is found. Finally, an outlook on laminar chaos in systems with state-dependent delay is given. I acknowledge the contributions of the late Günter Radons to these results.

DY 8.10 Mon 18:00 H43

**Chaotic Diffusion in Systems with Delay** — •Tony Albers, David Müller-Bender, and Lukas Hille — Institute of Physics, Chemnitz University of Technology, Chemnitz, Germany

Chaotic Diffusion is a purely deterministic phenomenon occurring in nonlinear systems. While much is known about chaotic diffusion in low-dimensional dynamical systems such as iterated maps or Hamiltonian systems, there are only a few works dealing with chaotic diffusion in higher-dimensional systems. In this talk, we show that chaotic diffusion is also possible in dynamical systems with time delay, which raises the dimension of the problem formally to infinity. Moreover, we demonstrate that introducing a periodic modulation of the delay can increase the strength of the diffusion, as measured by the diffusion coefficient, by several orders of magnitude [1]. This phenomenon is counterintuitively related to a significant decrease of the Kaplan-Yorke dimension of the chaotic attractor due to the appearance of so-called laminar chaos [2], which is a recently discovered type of chaos that is not observed in systems with constant delay. We acknowledge the contributions of the late Günter Radons who initiated this work.

[1] Tony Albers, David Müller-Bender, Lukas Hille, and Günter Radons, Phys. Rev. Lett. **128**, 074101 (2022)

[2] David Müller, Andreas Otto, and Günter Radons, Phys. Rev. Lett. **120**, 084102 (2018)

DY 8.11 Mon 18:15 H43

**Weak generalized synchronization in random neural networks and its impact on time series forecasting** — •hiromichi suetani[1,2] and ulrich parlitz[3,4] — [1]Faculty of Science and Technology, Oita University, Oita, Japan — [2]International Research Center for Neurointelligence , The University of Tokyo, Tokyo, Japan — [3]Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany — [4]Institute for the Dynamics of Complex Systems, Universität Göttingen, Göttingen, Germany

Time series forecasting is one of the important issues in data science, and approaches based on reservoir computing (RC), have been attracting attention. Previous studies have often suggested that the hyperparameter region at the so-called “edge of chaos,” provides optimal performance in time series forecasting. But this concept is problematic because generally a reservoir is a non-autonomous dynamical system driven by external inputs, it should be referred to as the “edge of conditional stability” rather than the edge of chaos.

In this study, we argue that this is not just a matter of terminology, and that the edge of conditional stability does not provide optimal performance. For this purpose, we clarify the relevance of the concept of “weak generalized synchronization (W-GS).” This study demonstrates that random neural networks driven by chaotic inputs exhibit W-GS and shows that the fractal nature of the GS function affects forecasting ability. We quantitatively compare the relationship between the characteristics of GS and those of RC, such as the information processing capacity, to elucidate the role of GS in RC.